Unit and dimension (L-2)

Q. What do you understand by dimensions. (defination)?

Ans. Dimensions of a physical quantity are the powers to which the basic quantity are raised to represent that quantity.

Eg. Force = mass × $\frac{\text{velocity}}{\text{time}}$ = mass × $\frac{\text{length}}{(\text{time})^2}$

$v = \frac{l}{t}$

$= [\text{mass}] \times \text{length} \times (\text{time})^{-2}$

$\Rightarrow$ the dimention of force are 1 in mass, 1 in length and -2 in time.

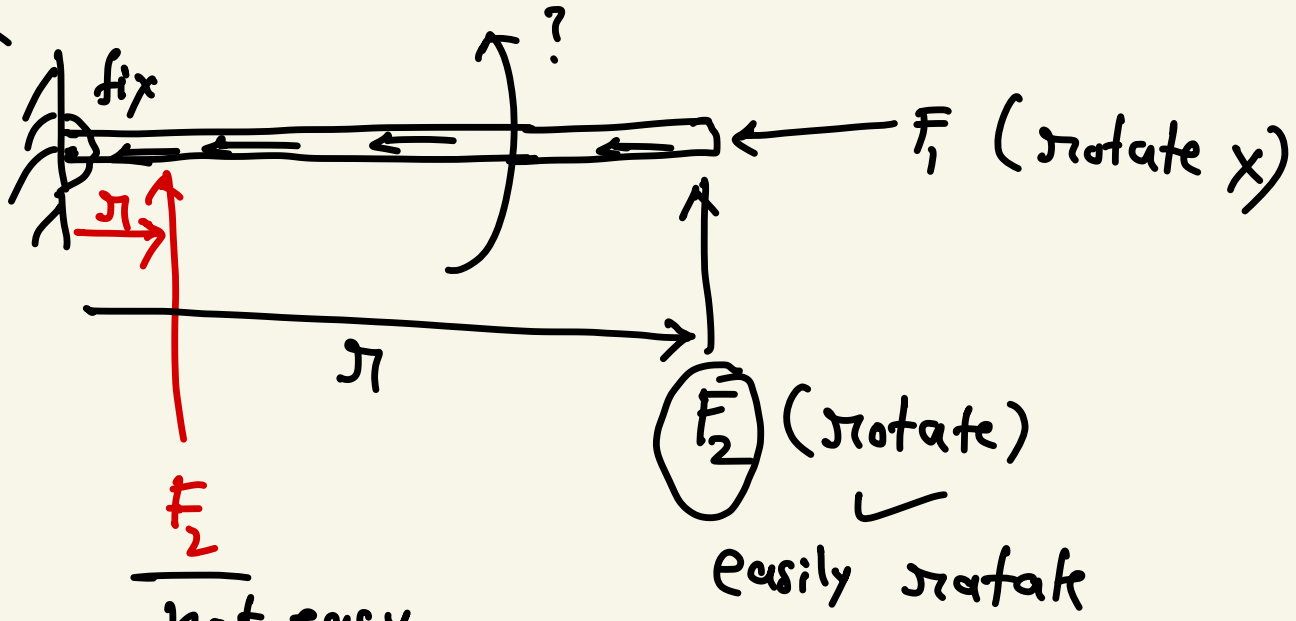
Q. Dimentional formula of torque?

$\tau \leftarrow$ (torque = force . distance.) $(\vec{\tau} = \vec{r} \times \vec{F})$

Sol. $F = M^1 L^1 T^{-2}$ & $dist = L^1$

$\tau = M^1 L^2 T^{-2}$

Torque



not easy to rotate

easily rotate

$$\tau = r F \sin\theta$$

Ex

(i) Gravitational const

$$F = \frac{G m_1 m_2}{r^2}$$

$$G = [M^{-1} L^3 T^{-2}]$$

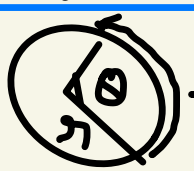
(ii) Surface tension.

$$= \left(\frac{F}{l}\right)$$

imp ★ Tension is a force $(M L^1 T^{-2})$

important point about dimension

(i) Angles & trigonometric f^n are dimensionless



$\star \left(\theta = \frac{l}{r}\right) \quad \frac{L^1}{L^1} = L^0$ (no-dim^n)

≡ * $\sin\theta$ = no dimension.

≡ $\sin(ax)$ $(\theta = ax)$ where a is const and x is length.

find d.F of a ?

Sol^n. $\sin(\theta = ax)$ → dimension less

means $ax = M^0L^0T^0$

$a(L^1) = M^0L^0T^0$

$$a = \frac{M^0L^0T^0}{L^1} = M^0L^{-1}T^0$$

$\boxed{a = L^{-1}}$ Ans.. (clear?)

≡ $\sin\left(\frac{kx}{t}\right)$ x → length, t → time. d.F of k ?

Sol^n.

$$\frac{k(L^1)}{T^1} = M^0L^0T^0 \Rightarrow \boxed{k = L^{-1}T^1M^0}$$

Ex

$\sin\left(\frac{a+x}{x_2}\right)$. $a \to$ (d. F) ?

Sol

length + mass = not possible

★ length + length = possible.

$x \to$ length. , a must be length.

$\boxed{a = L^1}$ Ans.

$\sin\left(\frac{a+x \to \text{length.}}{x_2 \to \text{length}}\right) = \text{dime}^{n}\text{. less.}$

(ii) All the exponents are dim^{n}-less.

e^{α}, e^{at^2} → no-dimension. find. d. F of a?

$(2.14)^{\text{meter}}$ meaningless.

Sol^{n}.

$at^2 = M^0L^0T^0$

$a(T^2) = M^0L^0T^0$

$\boxed{a = T^{-2}}$

(iii) Log function are dimension. less Ans.

$\log(a)$ $\boxed{a = M^0L^0T^0}$

$(\log(a.b) = \log a + \log b.)$

Application of dimensional analysis

1st :- To cheque the correction of an equation.

Eg $V = u + at$

a = acceleration
t → time
V & u are velocity

$\left(\frac{m}{s}\right) \quad \left(\frac{m}{s}\right) \quad \frac{m}{s^2} \times s = \left(\frac{m}{s}\right)$

Or

Find dimensional formula of A, B, C?

$$V = Ax + \frac{Bt}{C + At}$$

Velocity ←

x = distance
t = time.

Sol^n.

This must be velocity

$\frac{L^1}{T^1} = A \cdot (L^1)$

$T^{-1} = A$ Ans.

$At = (T^{-1})(T)$

$= T^0$

dimension of C will be same as At ⇒ T^0 Ans.

B = ? $\left[\frac{Bt}{(\text{no-dimension})} = \text{velocity} \rightarrow L^1T^{-1}\right]$

$$B(t) = L^1 T^{-1}$$

$$B(T^1) = L^1 T^{-1}$$

$$\boxed{B = L^1 T^{-2}}$$

Ex H.W. (i) $a = Ax^2 + Bt + ABC$

$a \rightarrow$ acceleration, $x \rightarrow$ distance, $t \rightarrow$ time.

find $[A]$, $[B]$, $[C]$.

Ans. $[A] = L^{-1} T^{-2}$

$[B] = L^1 T^{-3}$

$[C] = L T^3$ Check yourself.

(ii) $a = \sqrt{\pi A t}$ (π: dimensionless)

t = time

a = acceleration.

Ans. $[A] = L^2 T^{-5}$

(iii) $x = \frac{A}{B}(1 + e^{Bt})$

Ans $[A] = L^1 T^{-1}$

$[B] = L^{-1}$

(11) Second application. (unit - conversion)

Eg. length = 1 km = 1000 m = 1000,00 cm.

(n_1 = 1000, m → MKS system; n_2 = 1000,00, cm → CGS system.)

$$n_1 u_1 = n_2 u_2$$

units.

★ imp

★ Eg find value of 1 N in MKS in CGS - system?

$$n_1 u_1 = n_2 u_2$$

$$1\ N = n_2\ (u_2)$$

$F = ma$ (N)

$$1 \cdot \left(\frac{kg \cdot m}{s^2}\right) = n_2 \cdot \left(\frac{gm \cdot cm}{s^2}\right)$$

kg. m / s^2 → newton.

gm. cm / s^2 → dyne

$$\Rightarrow 1 \cdot \left(\frac{10^3\ gm \cdot 10^2\ cm}{s^2}\right) = n_2 \left(\frac{gm \cdot cm}{s^2}\right)$$

$n_2 = 10^5$

$1\ N = 10^5$ dyne